UNIVERSITÉ DE LAUSANNE
INSTITUT DE PHYSIQUE THÉORIQUE

**Séminaire interne
de physique théorique**

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Landauer model, representation theory and electrical resistance of high - scatterer density

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Landauer model, representation theory and the electrical resistance at high scatterer densities

1. Introduction
2. Model
3. How to compute $\langle g \rangle, \langle g^2 \rangle \ldots$?
   a) General case
   b) Pδ - model

Introduction

Aim: A simple quantum-mechanical model describing the main features of electrical transport in disordered matter.

Landauer [1970] reintroduces:
For determing the resistance of a sample we can first adjust the current flow and then look at the potential drop as vice versa.

In 1D the situation is as follows.

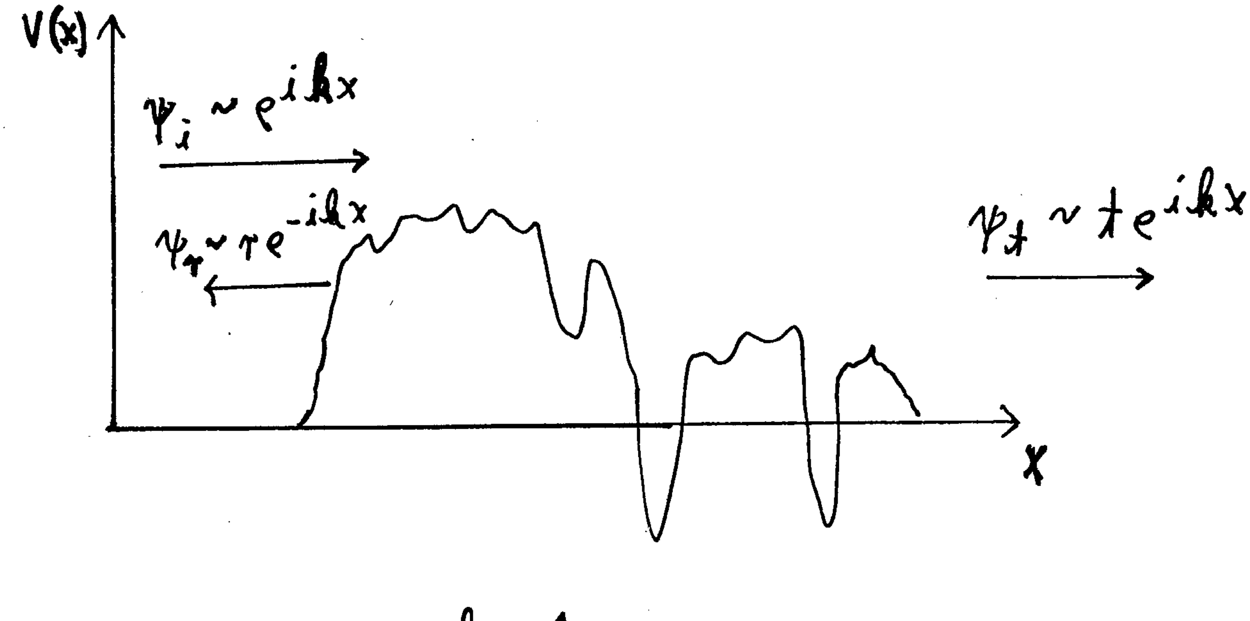

fig. 1

Landauer's result: $\underline{\underline{R \sim \frac{|r|^2}{|t|^2} := \varrho}}$

Model

δ-potentials distributed with Poisson statistics over fixed line segment [see Eberle, 1982, dissertation, and Feldorhof & Ford, 1986]

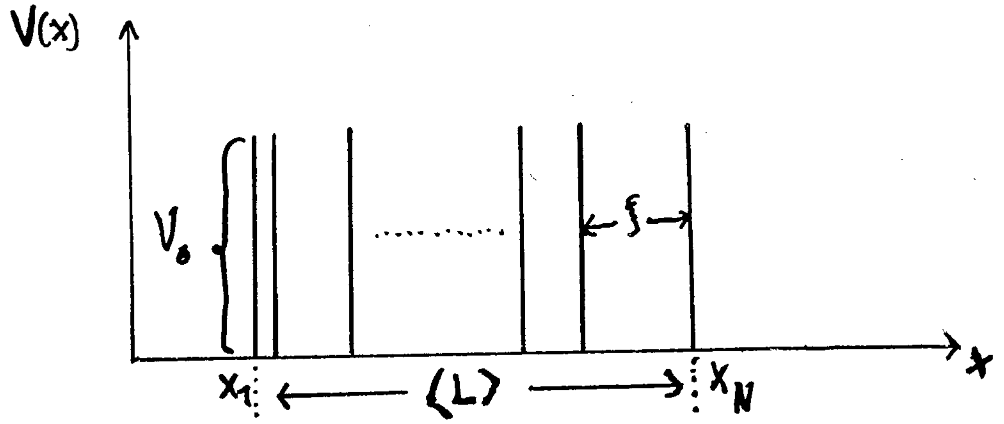

fig. 2

$$p(\xi) = n e^{-n\xi}$$

$$V(x) = V_0 \sum_{j=1}^{N} \delta(x - x_j)$$

$$\rho(L) = n e^{-nL} \frac{(nL)^{N-2}}{(N-2)!}$$

Properties of the length distribution

$$\boxed{\langle L \rangle = \frac{N-1}{n} \qquad \sigma^2 = \langle L^2 \rangle - \langle L \rangle^2 = \frac{\langle L \rangle}{n}}$$

Open and fixed ends models get as close as we want, if we take $\lim\limits_{\substack{n \to \infty \\ \langle L \rangle = \text{const}}}$ in the first one!

3. How to compute $\langle \rho \rangle, \langle \rho^2 \rangle \ldots\ldots$?

a) General case

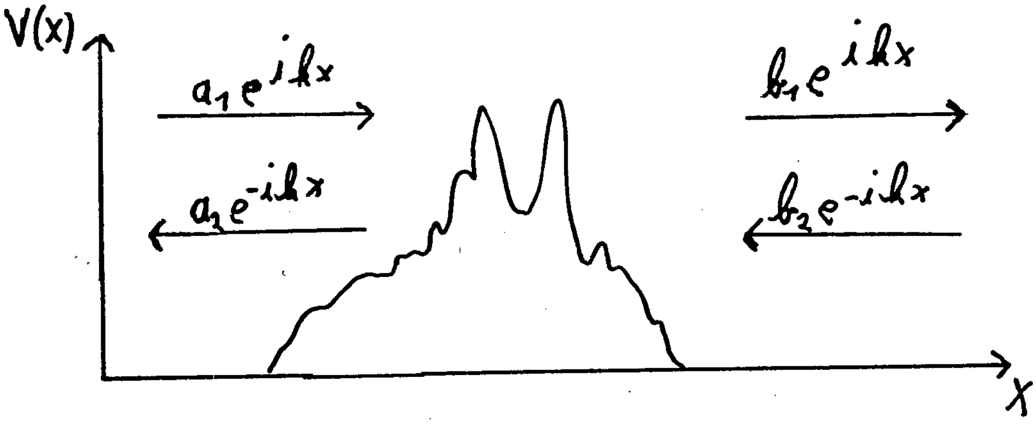

fig. 3

general solution:

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = R \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \;; \qquad R = \begin{pmatrix} a & b^* \\ b & a^* \end{pmatrix}$$

scattering solution:

$$\begin{pmatrix} t \\ 0 \end{pmatrix} = R \begin{pmatrix} 1 \\ r \end{pmatrix} \qquad R = \begin{pmatrix} \frac{1}{t^*} & -\frac{r^*}{t^*} \\ -\frac{r}{t} & \frac{1}{t} \end{pmatrix}$$

N - scatterers:

$$R_G = R_N R_{N-1} \cdots R_2 R_1$$

Extraction of the interpotential distance

$$R_G = K_N G_{N-1} K_{N-1} G_{N-2} \cdots G_2 K_2 G_1 K_1$$

$$G_j = \begin{pmatrix} e^{ik\xi_j} & 0 \\ 0 & e^{-ik\xi_j} \end{pmatrix} \; ; \; \xi_j = x_{j+1} - x_j$$

Averaging leads to

$$\langle R_G \rangle = \begin{pmatrix} \langle \frac{1}{t^*} \rangle & \langle -\frac{r^*}{t^*} \rangle \\ \langle -\frac{r}{t} \rangle & \langle \frac{1}{t} \rangle \end{pmatrix} = \langle G \rangle^{-1} \left(\langle G \rangle \langle K \rangle \right)^N$$

But we want $\langle \frac{|r|^{2k}}{|t|^{2k}} \rangle$; $k \in \mathbb{N}$!

3D symmetric representation

general solution:

$$\begin{pmatrix} b_1^2 \\ b_1 b_2 \\ b_2^2 \end{pmatrix} = \begin{pmatrix} a^2 & 2ab & b^{*2} \\ ab & (|a|^2+|b|^2) & a^* b^* \\ b^2 & 2a^* b & a^{*2} \end{pmatrix} \begin{pmatrix} a_1^2 \\ a_1 a_2 \\ a_2^2 \end{pmatrix}$$

scattering solution:

$$\begin{pmatrix} t^2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & 1+2g & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \begin{pmatrix} 1 \\ r \\ r^2 \end{pmatrix}$$

conclusion:

$$\boxed{\langle g \rangle = \frac{1}{2} \left(\langle {}^3G \rangle \langle {}^3K \rangle \right)^N_{00} - \frac{1}{2}}$$

5D symmetric representation

scattering solution:

$$\begin{pmatrix} t^4 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1+6\rho+6\rho^2 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} \begin{pmatrix} 1 \\ r \\ r^2 \\ r^3 \\ r^4 \end{pmatrix}$$

conclusion:

$$\boxed{\langle \rho^2 \rangle = \frac{1}{6} \left(\langle {}^5G \rangle \langle {}^5K \rangle \right)^N_{00} - \langle \rho \rangle - \frac{1}{6}}$$

b) PS-model

$$\langle {}^2G \rangle = \begin{pmatrix} \frac{\nu}{\nu-i} & 0 \\ 0 & \frac{\nu}{\nu+i} \end{pmatrix} \xrightarrow[\nu\to\infty]{} \begin{pmatrix} 1+\frac{i}{\nu} & 0 \\ 0 & 1-\frac{i}{\nu} \end{pmatrix} + {}^2O\left(\frac{1}{\nu^2}\right)$$

$$\nu = \frac{n}{k}$$

$$\langle {}^3G \rangle = \begin{pmatrix} \frac{\nu}{\nu-2i} & & \\ & 1 & \\ & & \frac{\nu}{\nu+2i} \end{pmatrix} \xrightarrow[\nu\to\infty]{} \begin{pmatrix} 1+\frac{2i}{\nu} & & \\ & 1 & \\ & & 1-\frac{2i}{\nu} \end{pmatrix} + {}^3O\left(\frac{1}{\nu^2}\right)$$

Eyh! What's that?

$$\boxed{\langle {}^3G \rangle \underset{\nu\to\infty}{=} {}^3\langle {}^2G \rangle + {}^3O\left(\frac{1}{\nu^2}\right)}$$

This is shock treatment! It's also

$$\boxed{\langle {}^5G \rangle \underset{\nu\to\infty}{=} {}^5\langle {}^2G \rangle + {}^5O\left(\frac{1}{\nu^2}\right)}$$

What does this mean?

The problem is solved to first order, because it can be reduced to a 2×2 matrixe.

Without computation we can conclude

$$\boxed{\begin{aligned} \langle \rho^N \rangle &= \left| \left\langle \frac{r}{t} \right\rangle \right|^{2N} + hot \\ \sigma^2 &= \frac{\langle \rho^2 \rangle - \langle \rho \rangle^2}{\langle \rho \rangle^2} = 0 + hot \end{aligned}}$$

hot: higher order terms

Tremendous task of diagonalizing a 2×2 matrix yields

$$\boxed{\langle g \rangle \underset{\substack{\nu\to\infty \\ (N\to\infty)}}{=} \frac{\beta N}{2k\langle L\rangle} \sinh^2\left(\sqrt{2\beta k \langle L\rangle N}\right) + \text{hot}}$$

$$\nu = \frac{N}{k\langle L\rangle} ; \quad \beta = \frac{m V_0}{\hbar^2 k}$$

which is exactly the value of the "optical potential" (rectangular barrier) as already supposed by Felderhof and Ford (1986), who found the exponential for $\beta > 0$.

The growth of $\langle g \rangle$ with $e^{a\sqrt{N}}$ (for $\beta > 0$) also coincides with numerical studies by Eberle & Erdös (1981).